Invited - A Planar Negative-Refractive-Index Medium Without Resonant Elements

Arthur A. Oliner

Polytechnic University, 11 Dawes Road, Lexington, MA 02421 USA

Abstract -- **Until now, structures that exhibited negative-refractive-index properties were 3D in form and employed resonant elements. It is widely believed that such resonant elements are essential. The present paper introduces two new features: *non-resonant elements* in the periodic array, and the adaptation of these ideas to *planar* structures, such as strip line or microstrip line.**

I. INTRODUCTION

Much interest has been expressed recently in a new class of periodic structures that can act as a medium that possesses an effective negative refractive index. These structures have also been characterized as metamaterials, as backward-wave materials (since the phase and group velocities point in opposite directions), as left-handed materials, as negative-refraction materials, and as double-negative materials. This class of periodic structures can properly be designated by any or all of these names (since they each describe a different aspect), but the term double-negative (DNG), introduced by Ziolkowski, seems to be the only one, aside from negative refraction, that is unique to this class. It indicates that the periodic structure is composed of pairs of elements, in which an array of one of the elements by itself produces an effective negative dielectric constant and an array of the other element corresponds to an effective negative permeability. A proper combination of the two arrays results in a negative product, yielding a backward-wave medium corresponding to a negative refractive index.

The present paper introduces two new features: ***non-resonant elements*** in the periodic array, and the adaptation of these ideas to ***planar*** structures, such as strip line or microstrip line.

The first of these new features responds to the fact that the structure in this class that has been analyzed[1] and measured[2] utilizes a resonant element (a split-ring configuration) for the effective negative permeability. This has led to the widespread perception that the array of periodic elements must contain a resonant constituent. The present paper describes an example of a double-negative array of elements that does not contain ***any*** resonant elements. It consists simply of element pairs comprised of an inductive vertical metal rod (or strip), together with a simple capacitive series gap. It is surely one of many possible examples, but it has the virtue of employing very simple elements, characterizable by simple but accurate analytical expressions.

The second new feature is an extension of these ideas to planar structures in order to permit their application to microwave integrated circuits. The structures treated here are based on strip line because simple and accurate expressions are available for both the rod and gap elements, which respectively produce the effective negative values of permittivity and permeability.

These strip-line structures grew naturally out of the earlier studies described above, but very recent work shows them to be examples of the LC transmission-line networks proposed independently by groups at the University of Toronto[3] and UCLA[4], of which the Toronto work was presented earlier. The work described in this paper is different from theirs because it employs analytical expressions to derive the specific performance characteristics in a direct way.

II. VARIOUS POSSIBLE STRUCTURES FOR A PLANAR MEDIUM WITH NEGATIVE REFRACTIVE INDEX

As mentioned in the Introduction, the class of structures considered here is based on a double-negative periodic array of elements that do ***not*** contain any resonant elements. It consists simply of element pairs comprised of an inductive vertical metal rod (or strip) and a simple capacitive series gap.

In order to achieve the series capacitive gap, the proposed structure is arranged in ***planar*** form. The two most familiar 1D configurations are ***strip line*** and ***microstrip line***, which are sketched roughly in Fig.1. With minor modifications in the structures, they can be made suitable for 2D versions, with planar waves incident at an angle.

0-7803-7695-1/03/$17.00 © 2003 IEEE

The individual elements are vertical rods or strips placed parallel to the electric field, and series capacitive gaps located in the strips of these transmission lines. The individual elements can be ***paired*** in the two different ways shown in Fig.2. One way is to locate the vertical rod in the middle of the gap, and the other is to place a rod on each side of the gap. The details relating to the properties of the individual elements are discussed in Sec. III.

For either microstrip line or strip line, the gap is made in the guiding strip. For microstrip line, the rods extend only in the region of the dielectric substrate, between the ground plane and the bottom of the guiding strip, whereas for strip line the rods extend in both the upper and lower portions of the line, where, we recall, the electric fields are oppositely directed in the two halves of the line.

The arrangements for strip line or microstrip line described above apply directly for 1D guidance. By simple modifications in the configurations (and in the analytical expressions for the rod and gap elements), the same basic considerations can be applied to a variety of other situations. For example, parallel arrays of strip line can be arranged, with a wide planar beam incident at any angle, as shown in Fig. 3, or various actual 2D structures can be designed. A number of such configurations were presented at a recent symposium [5].

III. THE CONSTITUENT ARRAYS

To exhibit a negative refractive index, a periodic medium must consist of two independent periodic arrays that are combined together. One of these arrays must provide the equivalent of a negative permittivity and the other a negative permeability. How these are combined, and what propagation properties follow from their combination, are discussed in Sec. IV. In this section, we describe their geometries, equivalent networks, and independent dispersion behaviors.

A. Array of Vertical Strips

As pointed out in the Introduction, a periodic array of vertical rods or strips, oriented parallel to the electric field, exhibits a stop band that ranges from zero frequency up to a frequency determined by the detailed geometry of the array. Within that stop band, the effective permittivity is negative. These properties have been recognized for many years, and have been used in recent earlier studies such as the key ones in [1, 2].

The array of vertical strips used in the planar structure described in Fig.3 is shown in both front and top views in Figs. 4(a) and 4(b), respectively. The complete 2D array consists of a 1D array in the xy plane, which in turn is an element of the array of these in the z direction, which is the propagation direction. A front view of this 1D array is seen in Fig. 4(a), which shows that the individual strips in that 1D array have a strip width equal to δ, and are spaced a apart in the x direction. The top view in Fig. 4(b) indicates additionally that the individuals strips are very thin, and that the incoming wave need not be normally incident on the array ($\theta = 0$), but can be incident at any angle.

The equivalent network of this 1D array for propagation in the z direction is a simple shunt inductive element, X, as shown in Fig. 4(c). If the complete array were a 1D rather than a 2D array, only a single strip would be present in Figs. 4(a) and 4(b), but, of course, the form of the network in Fig. 4(c) would remain the same.

An accurate expression for the inductive reactance X/Z_0 in Fig. 4(c) may be found in the Waveguide Handbook [6], in Sec. 5.19, pp. 284-285. It is not necessary to use the complicated expression (1a) unless the parameters approach the edges of the ranges of validity of the expression. Instead, for most values, one can employ (1b) or even a simplification of it, so that one can obtain a rather simple final expression for X/Z_0 that is still reasonably accurate. This simple expression after changing the notation to be consistent with Figs. 4, is

$$X/Z_0 = (a \cos\theta / \lambda_0)\,[\ln \csc (\pi\delta/2a)] \qquad (1)$$

B. Array of Horizontal Gaps

A side view of an array of horizontal gaps is shown in Fig. 5(a). The gap spacing is s, the period in the z direction is d, the spacing between the top and bottom plates is b, and the electric field directions in the upper and lower halves are opposite to each other, as required for strip-line performance.

The equivalent network for an individual gap in the strip-line geometry shown in Fig. 5(a) has the form seen in Fig. 5(b). It is a series capacitive element if the gap is relatively narrow, which is what we want anyway. If the gap becomes wide, then shunt inductive elements must be included and the network becomes a pi network.

Fortunately, a fairly accurate but simple expression for the susceptance of the capacitive gap is already available. It was derived as part of a group of various discontinuity structures in strip line [7,8]. Expressions are given there for both the series and shunt elements of the complete pi network, but only the result for the series element will be presented here because the shunt elements can be neglected in the range of small s/b values. The expression is

$$B/Y_0 = (b \cos\theta/\lambda_0)\ [\ln \csc (\pi s/2b)] \qquad (2)$$

where the factor cos θ has been added to take into account incidence at an angle. Expression (2) is accurate over a large range of values of s/b provided that b is sufficiently smaller than the wavelength, which is generally the case for strip line. The expression has been found to agree well with measured values taken by H. Keen of the Airborne Instruments Laboratory [8].

C. Band-Structure Plots (Dispersion Curves) for Arrays of the Constituent Elements

We next consider the band-structure or dispersion behavior for the arrays of the constituent elements ***separately***. A longitudinal (side) view of a periodic array of either ***single vertical strips***, or ***an array of them*** as seen in Figs. 4(a) and 4 (b), is shown in Fig. 6(a). The period in the z direction is seen to be d, and an equivalent network for a unit cell of this array, from $-d/2$ to $d/2$, appears in Fig. 6(b).

It is known from periodic structure theory that simple expressions can readily be derived for the locations of the ***band edges*** in a band-structure plot. These plots, of k_0d vs. βd, where k_0 is the free-space wavenumber and β is the longitudinal propagation wavenumber or phase constant, possess successive pass bands and stop bands. The boundaries between them are the band edges. It can be shown that, for symmetrical unit cells, these band edges can be calculated from the resonances of a half unit cell. Since the space available here is limited, we shall present only the results for the band structure.

The band-structure plot for a typical set of geometrical parameter values for the array of vertical strips alone is shown in Fig. 7. The most important feature is that a stop band begins at zero frequency and, depending on the geometrical parameters, can remain a stop band up to close to $k_0d = \pi$. The pass band then ends at $k_0d = \pi$, where the next stop band begins.

A side view, or longitudinal view, of the ***array of horizontal gaps*** was shown earlier as Fig. 5(a), where the spacing between successive gaps is the period d. An equivalent network for a unit cell would then be the same as the one in Fig. 6(b) for the array of strips except that the shunt inductive element X would be replaced by the series capacitive element B, and that Z_0 would be replaced everywhere by Y_0. Furthermore, a comparison between expressions (1) for the strips and (2) for the gaps shows that they are very similar, with a instead of b and δ instead of s. In fact, the array of gaps is actually a dual of the array of strips.

When one derives the expressions for the band edges for the array of gaps, the results obtained are the exact dual of those for the strips. Of course, the widths of the stop bands will not be identical since they depend on the specific numerical values for the geometrical parameters.

IV. COMBINING THE VERTICAL STRIPS AND HORIZONTAL GAPS

As pointed out in Sec. II, there are two different ways in which the vertical inductive strips and the horizontal capacitive gaps can be combined as a paired single discontinuity element. These two different ways are shown in Fig. 2. One way is to locate the vertical strip (or rod) in the middle of the gap, and the other way is to place a strip on each side of the gap. Both ways of pairing the constituent discontinuities have drawbacks. After assessing these drawbacks it was decided that this paper should adopt the second way, placing a strip on each side of the gap.

The combined discontinuity element is now shown in Fig. 8(a), and its bisected equivalent network is given in Fig. 8(b). As with the constituent discontinuities discussed in Sec. III, expressions were derived for the band edges of the band-structure plot. The width of the band that yields the negative-index property can be large or small depending on the specific parameter values of the constituent discontinuities.

The band-structure plot for a particular set of dimensional parameters is shown in Fig. 9. The structure is seen to support a backward wave, as required, so that the wave possesses a positive group velocity and a negative phase velocity.

References

[1] J. B. Pendry, A. J. Holden, D. R. Robbins and W.J. Stewart, "Magnetism from Conductors and Enhanced Nonlinear Phenomena," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-47, pp. 2075-2084, Nov. 1999.

[2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity," *Phys. Rev. Lett.*, vol. 84, pp. 4184-4187, 1 May 2000.

[3] G. V. Eleftheriades, A. K. Iyer and P. C. Kremer, "Planar Negative Refractive Index Media Using Periodically L-C Loaded Transmission Lines", *IEEE Trans. Microwave Theory Tech.*, vol. 50, pp. 2702-2712, Dec. 2002.

[4] C. Caloz, H. Okabe, T. Iwai and T. Itoh, "Transmission Line Approach of Left-Handed Materials," URSI Digest, *IEEE AP-S/URSI International Symp.*, June 16-21, 2002, San Antonio, TX.

[5] A. A. Oliner, "A Periodic-Structure Negative-Refractive-Index Medium Without Resonant Elements," URSI Digest, *IEEE AP-S/URSI International Symp.*, June 16-21, 2002, San Antonio, TX.

[6] N. Marcuvitz, *Waveguide Handbook*, McGraw-Hill, NY, 1951. (Vol. 10 of MIT Rad. Lab. Series)

[7] A. A. Oliner, "Equivalent Circuits for Discontinuities in Balanced Strip Transmission Line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 134-143, March 1955.

[8] H. M. Altschuler and A. A. Oliner, "Discontinuities in the Center Conductor of Symmetric Strip Transmission Line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 328-339, May 1960.

[This Digest paper has 9 figures, but the website for the IMS Digest consistently declined to upload them. I therefore reluctantly decided to submit the text without the figures.]

Figure Captions

Fig. 1 Best-known planar 1D transmission lines.

Fig. 2 Top view of two different ways in which the individual elements can be paired.

Fig. 3 An array of strip lines or microstrip lines with a wide planar beam incident at some angle. The period in the longitudinal direction z is d.

Fig, 4 Front view, top view, and equivalent network of the array of vertical strips shown in Fig. 3.

Fig. 5 Side view of an array of horizontal gaps in strip-line, and the equivalent network for a single gap.

Fig. 6 Longitudinal (side) view of a periodic array of either single vertical strips, or an array of them, together with an equivalent network for a unit cell of this array.

Fig. 7 Band-structure plot for a typical set of parameters for the array of vertical strips alone.

Fig. 8 Side view of the combined pair of discontinuity elements, and its bisected equivalent network.

Fig. 9 Band-structure plot for the array of paired discontinuity elements shown in Fig. 8. A backward wave is produced, as required.